

Vector-Chiral Equivalence in Null Gauged WZNW Theory

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Abstract

We consider the standard vector and chiral gauged WZNW models by their gauged maximal null subgroups and show that they can be mapped to each other by a special transformation. We give an explicit expression for the map in the case of the classical Lie groups A_N , B_N , C_N , D_N , and note its connection with the duality map for the Riemannian globally symmetric spaces.

1 Introduction

Duality as a abelian or non-abelian symmetry of conformal field theories has been studied extensively in the last few years [1],[2],[3]. In WZNW models, this symmetry, when originating from an abelian subgroup of the isometry of the conformal background, has been better understood.

In fact, it has been shown that automorphisms of the group G of the model are responsible for their duality transformations [4]. Specifically, it has been proved that the target manifold of the WZNW model with the group G , obtained by vector gauging its abelian subgroup H , is dual to the manifold obtained by axially gauging the same subgroup [5]; and this dual transformation is implemented by an automorphism of H , on the left action of H on G .

The above considerations are generally valid, whether the subgroup H is semisimple or not. But, it has been known that when H is not semisimple, certain new phenomena could occur. In particular when H is nilpotent,¹ the target space shows unusual features and its dimension reduces unexpectedly [6], [7],[8].

In other words, it was shown that in the usual vector gauging of $SL(2, R)$ WZNW model, by its nilpotent one dimensional subgroup, the target space metric becomes one dimensional and the effective action of the gauged model reduces to that of the Liouville field theory. Similar behaviour for $SO(3, 1)$ vector gauged WZNW, by its two dimensional Euclidean subgroup, is also observed [9]. In this case, although we expect a three dimensional target space, the resultant target space is only one dimensional.

It has been suspected that the reduction of the degrees of freedom of the target manifold

¹To be exact one should consider null groups, which are slightly more general than the nilpotent groups.

for those coset G/H WZNW models, when a nilpotent subgroup H is vectorially gauged, is related to the obvious dimensional reduction of the corresponding chiral gauged model when the left and right actions are independent and in two different subgroups isomorphic to H [10].

This dimensional reduction occurs for the Toda theories by gauging the WZNW model, with the total left and right null subgroups G_+ and G_- generated by positive and negative step operators in the Cartan-Weyl decomposition of Lie group G . On the other hand, if the dimension of gauged subgroups are less than the dimension of G_+ and G_- , then in the resulting σ -model, one obtains the Toda theory with an interaction term that comes from the remaining part of ungauged directions of G_+ and G_- [11]. The Toda structure is also obtained, however, by simply gauging the nilpotent subgroup vectorially, in the case of $SO(3,1)$ [9].

In this letter we will construct a one to one map from the vector gauged model to the chiral gauged model G/H , when H is nilpotent, explaining the dimensional reduction of the vector gauged model and find that this map is in fact an automorphism of G . This automorphism turns out to be the dual map of the well known Riemannian globally symmetric manifolds.

In section two, we will briefly recall the construction of the gauged WZNW models, and describe the reduction of the dimension of the target manifold for $SL(2, R)$ by its nilpotent subgroup when gauging it vectorially. In section three we construct the automorphism relating the vector gauged WZNW model to the chiral case, and generalize this automorphism to the case of other simple Lie groups gauged by their maximal null subgroups. In section four, we note on the relation of this map to the corresponding duality map for Riemannian globally symmetric manifolds.

2 Structure of gauged WZNW models

Let us recall first the structure of vector and chiral gauged WZNW . The G/H vector gauged WZNW action [12], [13]

$$S_V(g, \mathbf{A}, \bar{\mathbf{A}}) = S(g) + \frac{k}{2\pi} \int d^2z \text{Tr}(-\bar{\mathbf{A}}g^{-1}\partial g + \mathbf{A}\bar{\partial}gg^{-1} - \mathbf{A}\bar{\mathbf{A}} + g\bar{\mathbf{A}}g^{-1}\mathbf{A}), \quad (2.1)$$

$$S(g) = \frac{k}{4\pi} \int d^2z \text{Tr}(g^{-1}\partial gg^{-1}\partial g) - \frac{k}{12\pi} \int \text{Tr}(g^{-1}dg)^3,$$

is invariant under the gauge transformations

$$g \rightarrow h^{-1}gh, \mathbf{A} \rightarrow h^{-1}(\mathbf{A} - \partial)h, \bar{\mathbf{A}} \rightarrow h^{-1}(\bar{\mathbf{A}} - \bar{\partial})h, \quad (2.2)$$

where $h = h(z, \bar{z})$ is a group element in subgroup H and \mathbf{A} , $\bar{\mathbf{A}}$ take their values in the algebra $\mathcal{L}(H)$ of the subgroup H . Parametrising \mathbf{A} and $\bar{\mathbf{A}}$ in terms of the group elements l , \bar{l} of H

$$\mathbf{A} = \partial l l^{-1}, \quad \bar{\mathbf{A}} = \bar{\partial} \bar{l} \bar{l}^{-1},$$

one can write the above form of vector action (2.1), using the Polyakov-Wiegmann identity [14], as the difference of two terms [15]

$$S_V(g, \mathbf{A}, \bar{\mathbf{A}}) = S(l^{-1}g\bar{l}) - S(l^{-1}\bar{l}). \quad (2.3)$$

Each term is invariant under gauge transformations, $g \rightarrow h^{-1}gh, l \rightarrow h^{-1}l, \bar{l} \rightarrow h^{-1}\bar{l}$. On the other hand, the chiral WZNW action [16], [17], [18]

$$S_C(g, \mathbf{A}, \bar{\mathbf{A}}) = S(g) + \frac{k}{2\pi} \int d^2z \text{Tr}(-\bar{\mathbf{A}}g^{-1}\partial g + \mathbf{A}\bar{\partial}gg^{-1} + g\bar{\mathbf{A}}g^{-1}\mathbf{A}), \quad (2.4)$$

is invariant under the following transformations

$$g \rightarrow h^{-1}g\bar{h}, \mathbf{A} \rightarrow h^{-1}(\mathbf{A} - \partial)h, \bar{\mathbf{A}} \rightarrow \bar{h}^{-1}(\bar{\mathbf{A}} - \bar{\partial})\bar{h}, \quad (2.5)$$

where $h = h(z)$ belongs to the subgroup H_1 , and $\bar{h} = \bar{h}(\bar{z})$ belongs to another subgroup H_2 of G . \mathbf{A} takes its value in $\mathcal{L}(H_1)$ and $\bar{\mathbf{A}}$ in $\mathcal{L}(H_2)$. Parametrising $\mathbf{A}, \bar{\mathbf{A}}$ in terms of $l \in H_1$ and $\bar{l} \in H_2$ as before, $S_C(g, \mathbf{A}, \bar{\mathbf{A}})$ can be written as

$$S_C(g, \mathbf{A}, \bar{\mathbf{A}}) = S(l^{-1}g\bar{l}) - S(l^{-1}) - S(\bar{l}).$$

We see that these chiral transformations don't fix and eliminate any dynamical degree of freedom of g , since h and \bar{h} are only holomorphic and antiholomorphic functions respectively and not arbitrary functions of z and \bar{z} , in contrast to vector case. But if we restrict H_1 and H_2 to be null or nilpotent subgroups of G (A group H is null if $Tr(N_i^2) = 0$, for every i where N_i 's are generators of algebra $\mathcal{L}(H)$; and nilpotent if $N_i^2 = 0$) then $S(l^{-1}) = S(\bar{l}) = 0$ and $S_C(g, \mathbf{A}, \bar{\mathbf{A}}) = S(l^{-1}g\bar{l})$ is invariant under truly gauge transformations (2.5) with $h = h(z, \bar{z}) \in H_1$ and $\bar{h} = \bar{h}(z, \bar{z}) \in H_2$ or $g \rightarrow h^{-1}g\bar{h}, l \rightarrow h^{-1}l, \bar{l} \rightarrow \bar{h}^{-1}\bar{l}$. In the following we choose $H_1 = G_+$ (in the vector case we also choose $H = G_+$) and $H_2 = G_-$ respectively, where G_+ (G_-) is the null subgroup of G generated by the set of positive (negative) step operators.

Now, let us consider $SL(2, R)/E(1)$ vector gauged WZNW model. The $E(1)$ subgroup is generated by the nilpotent positive step operator $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ in the triangular decomposition of the Lie algebra $sl(2, R)$. The gauge part of the vector gauged action $S_V(g, \mathbf{A}, \bar{\mathbf{A}})$ with the parametrization of $g = \begin{pmatrix} a & u \\ -v & b \end{pmatrix}, ab + uv = 1$, is

$$\frac{-k}{\pi} \int d^2z \{ \bar{A}(v\partial a - a\partial v) + A(b\bar{\partial} v - v\bar{\partial} b) + v^2 A\bar{A} \}, \quad (2.6)$$

while the gauge part of the chiral gauged action $S_V(g, \mathbf{A}, \bar{\mathbf{A}})$, when G_+ and G_- are generated

by $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ and $\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$, is

$$\frac{-k}{\pi} \int d^2z \{ \bar{A}(b\partial u - u\partial b) + A(b\bar{\partial}v - v\bar{\partial}b) - b^2 A \bar{A} \}. \quad (2.7)$$

The common part $S(g)$ of the action for both cases is

$$\begin{aligned} & \frac{k}{2\pi} \int d^2z \{ b^2 \partial a \bar{\partial} a + bu(\partial a \bar{\partial} v + \partial v \bar{\partial} a) + u^2 \partial v \bar{\partial} v + bv(\partial u \bar{\partial} a + \partial a \bar{\partial} u) \\ & - ab(\partial u \bar{\partial} v + \partial v \bar{\partial} u) - uv(\partial b \bar{\partial} a + \partial a \bar{\partial} b) + au(\partial b \bar{\partial} v + \partial v \bar{\partial} v) + v^2 \partial u \bar{\partial} u \\ & + av(\partial u \bar{\partial} b + \partial b \bar{\partial} u) + a^2 \partial b \bar{\partial} b + \int \frac{da}{a} (\partial u \bar{\partial} v - \partial v \bar{\partial} u) \}. \end{aligned}$$

The term involving $\ln a$ comes from the Wess-Zumino term and can be written in the equivalent form $\ln u(\partial a \bar{\partial} b - \partial b \bar{\partial} a)$ [19]. The gauge symmetry (2.2) forces us to choose $a = 0$ or $b = 0$, in any case upon integration of the action over gauge fields A, \bar{A} , the resultant effective action leads to a target space metric and a dilaton field that are proportional to $d\phi^2$ and ϕ respectively, where $\phi = \ln v$ (The antisymmetric tensor is zero in this gauge). Obviously one dynamical degree of freedom has evaporated. A similar feature is observed for $SO(3,1)$ when vector gauging it by its three dimensional Euclidean group [9]. In this case two generators of $E(2)$ group that correspond to translation in two dimensional plane are null and the resultant effective action and dilaton field are one dimensional.

3 Equivalence of vector and chiral gauging

In the last section we saw that in vector gauging a WZNW model by a subgroup while is null (or when a part of the subgroup is null) a dimensional reduction in the effective action of the theory occurred. To explain this reduction, we consider the $SL(2, R)$ case first. It is

not hard to see that the following map changes the vector gauged action (2.6) to the chiral gauged action (2.7)

$$\begin{aligned} a &\rightarrow -u, u \rightarrow a, v \rightarrow b, b \rightarrow -v, \\ A &\rightarrow A, \bar{A} \rightarrow -\bar{A}, \end{aligned} \tag{3.1}$$

or in the compact form $g \rightarrow g\tau_2, \mathbf{A} \rightarrow \mathbf{A}, \bar{\mathbf{A}} \rightarrow \tau_2^{-1}\bar{\mathbf{A}}\tau_2$, where

$$\tau_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \tag{3.2}$$

Note that the $S(g)$ part is invariant under this map. Clearly, from the form of the action $g \rightarrow g\tau_2$ one can see that τ_2 is a symmetry of the vector gauged theory. On the other hand, τ_2 maps the vector gauged theory onto the chiral case, then establishing the equivalence of the two theories. But, in the case of the chiral gauge theory there is only one degree of freedom. In fact, from chiral symmetry (2.4), one can set $u = v = 0$. This, therefore explains the evaporation of the degree of freedom in the target space metric and dilaton of the vector gauged theory alluded to above [6],[7]. Note also that the target space metric is the free field part of the Toda-like action with its coordinates, the parameters of the Cartan part of the Gauss decomposition of the group element g ; and a linear sum of these parameters is the dilaton field [11].

In the part of this article we will attempt to generalize the above results to higher dimensional groups. We may repeat the same calculation for $SL(3, R)$ gauged WZNW model, G_+ subgroup is generated by the three positive triangular matrices $e_{ij}; j < i; i, j = 1, 2, 3$ with e_{ij} the matrix with one at the (i, j) entry and zero everywhere; similarly, the G_- subgroup is generated by $e_{ij}; j > i; i, j = 1, 2, 3$. We use the matrix representation of g of the form

$$\begin{pmatrix} a & b & c \\ d & e & f \\ h & i & j \end{pmatrix}. \quad (3.3)$$

Then a simple calculation shows that the vector gauged $SL(3, R)/G_+$ WZNW is converted to the chiral gauged $SL(3, R)$ with the following transformation on the $g, \bar{\mathbf{A}}$

$$a \rightarrow c, b \rightarrow b, c \rightarrow -a, d \rightarrow f, e \rightarrow e, f \rightarrow -d, h \rightarrow j$$

$$i \rightarrow i, j \rightarrow -h$$

$$\bar{A}_1 \rightarrow \bar{A}_3, \bar{A}_2 \rightarrow -\bar{A}_2, \bar{A}_3 \rightarrow -\bar{A}_1$$

or in the compact form as $g \rightarrow g\tau_3, \mathbf{A} \rightarrow \mathbf{A}, \bar{\mathbf{A}} \rightarrow \tau_3^{-1}\bar{\mathbf{A}}\tau_3$, where

$$\tau_3 = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}. \quad (3.4)$$

The vector gauged WZNW action is obviously invariant under these transformations and we therefore get the equivalence of the vector and chiral gauged theories. Generally, a similar result exists for $SL(2N, R)$. In other words, under the transformation

$$g \rightarrow g\tau_{2N}, \mathbf{A} \rightarrow \mathbf{A}, \bar{\mathbf{A}} \rightarrow \tau_{2N}^{-1}\bar{\mathbf{A}}\tau_{2N}, \quad (3.5)$$

the $SL(2N, R)$ vector gauged WZNW model converted to the chiral WZNW model, where

τ_{2N} is an off-diagonal matrix with N minus ones and N plus ones, $\tau_{2N} = \sum_{i=1}^N e_{i, 2N+1-i} - \sum_{i=N+1}^{2N} e_{i, 2N+1-i}$. Similarly for $SL(2N+1, R)$, the map is

$$\tau_{2N+1} = -\sum_{i=1}^N e_{i, 2N+2-i} + \sum_{i=N+1}^{2N+1} e_{i, 2N+2-i}. \quad (3.6)$$

Next, we consider the gauged WZNW actions with $SO(2N, R)$ group manifold. The first nontrivial case is $SO(4, R)$; the G_+ subgroup is generated by $e_{14} - e_{23}, e_{12} - e_{43}$, and G_- by $e_{32} - e_{41}, e_{21} - e_{34}$ [20], and the equivalence map is s_4 , with

$$g \rightarrow gs_4, \mathbf{A} \rightarrow \mathbf{A}, \bar{\mathbf{A}} \rightarrow s_4^{-1} \bar{\mathbf{A}} s_4, \quad (3.7)$$

where $s_4 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$. A similar but long calculation shows the same equivalence

between $SO(2N, R)$ vector and chiral gauged WZNW theories. In this case the transformations are like $SO(4, R)$ case (3.8), with s_4 replaced by s_{2N}

$$g \rightarrow gs_{2N}, \mathbf{A} \rightarrow \mathbf{A}, \bar{\mathbf{A}} \rightarrow s_{2N}^{-1} \bar{\mathbf{A}} s_{2N}, \quad (3.8)$$

where

$$s_{2N} = \sum_{i=1}^N e_{i, i+N} - \sum_{i=N+1}^{2N} e_{i, i-N}. \quad (3.9)$$

In a similar way the following operator s_{2N+1} relates the vector and chiral gauged theories with the group $SO(2N+1)$, where

$$s_{2N+1} = (-)^N \{e_{11} - (\sum_{i=2}^{N+1} e_{i, N+i} + \sum_{i=N+2}^{2N+1} e_{i, i-N})\}. \quad (3.10)$$

At last, let us consider the case of $SP(2N, R)$ gauged WZNW actions, the simplest being $SP(2, R)$. Here G_+ and G_- subgroups are exactly the same as in the $SL(2, R)$ case and the τ_2 matrix that of equation (3.2) relates the vector gauged theory to the chiral case. Similar results exist for $SP(2N, R)$ just as the transformations (3.9) and (3.10) for the $SO(2N, R)$ case.

4 Concluding Remarks

In the preceding sections, with the help of the operators $\tau_{2N}, \tau_{2N+1}, s_{2N}, s_{2N+1}$, we established the equivalence of the vector gauged action with that of the chiral gauged case. In fact, we learn from this equivalence that, when vector gauging WZNW by its null subgroup, in addition to the apparent symmetry (2.2), there is in another symmetry that is induced from the chiral gauged version and is as follows

$$g \rightarrow h' g h, \mathbf{A} \rightarrow h' (\mathbf{A} - \partial) h'^{-1}, \bar{\mathbf{A}} \rightarrow h^{-1} (\bar{\mathbf{A}} - \bar{\partial}) h, \quad (4.1)$$

where $h = h(z, \bar{z})$ and $h' = h'(z, \bar{z})$ are different group elements in the null subgroup $H = G_+$ of G .

The results obtained in the previous sections remain valid for the case that the gauge groups are subgroups of G_+ and G_- . Let us to call these gauge groups H_+ and H_- , with the constraint $H_- = \tau H_+ \tau^{-1}$, where τ is the appropriate involutive automorphism for G group that was constructed in the preceding sections. After integrating out the gauge fields here the σ -model analog of the Toda theory interacting with the remaining directions of G_+ and G_- that do not belong to H_+ and H_- respectively, is obtained [11]. Finally we would like to point out to an interesting connection between our equivalence map and the well known duality map of symmetric Riemmanian manifolds. Recall the way this duality maps appear in Riemmanian geometry [21]. For every Lie group G there are involutive automorphisms of the Lie algebra $\mathcal{L}(G)$ which have the additional property that convert the $\mathcal{L}(G_+)$ to $\mathcal{L}(G_-)$. There are only three different involutive automorphisms for every simple algebra $\mathcal{L}(G)$ up to conjugacy [22].

The complex conjugation operator C , the operator $I_{p,q}$ defined by $\begin{pmatrix} I_p & 0 \\ 0 & -I_q \end{pmatrix}$ and finally $J_{p,p} = \begin{pmatrix} 0 & I_p \\ -I_p & 0 \end{pmatrix} \simeq \tilde{J}_{p,p} = \begin{pmatrix} 0 & \tilde{I}_p \\ -\tilde{I}_p & 0 \end{pmatrix}$, where $I_p(\tilde{I}_p)$ is the $p \times p$ unit matrix with $+1$ on the major (minor) diagonal and 0 elsewhere. These involutive automorphisms was used to classify all real forms of the simple algebras [21]. In fact let g be a compact simple algebra and τ an involutive automorphism of g . Let $g = f \oplus p$ be the cartan decomposition of g into eigenspaces of the involutive automorphism τ , $\tau(f) = f, \tau(p) = -p$. If we now perform the Weyl unitary trick on the subspace p , we get a new algebra $g^* = f \oplus ip$, that is a real form of the complexification of g up to isomorphism. The resulting Riemannian globally symmetric spaces $G/F = EXP(p)$ and $G^*/F = EXP(ip)$ are said to be dual to each other [23].

As an example take g to be $su(2N, C)$ algebra (the compact real form of $SL(2N, C)$). With $\tilde{J}_{N,N}C$, the maximal compact subalgebra f and the noncompact algebra g^* are $usp(2N)$ and $su^*(2N)$, and the dual symmetric spaces are $SU(2N, C)/USP(2N)$, $SU^*(2N, C)/USP(2N)$. The operator $\tilde{J}_{N,N}$ is in fact the same operator that was used in section three to map the vector and chiral $SL(2N, R)$ WZNW to each other, $\tau_{2N} = \tilde{J}_{N,N}$.

Similarly, for $su(2N+1, C)$, the dual automorphism is $I_{N+1,N} = \tau_{2N+1}$ (up to conjugacy); for $so(2N)$, it is $J_{N,N} = s_{2N}$; and for $so(2N+1)$, we have $I_{1,2N} = (-)^N s_{2N+1}$ (up to conjugacy); and finally for $sp(2N, C)$ the correspondence is $J_{N,N} = s_{2N}$.

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